Asset Pricing Summary

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1 Testing CAPM

If CAPM holds, the market portfolio should have the maximum sharpe ratio. Equally, under the CAPM equation $R_{pt} = \alpha + \beta R_{mt} + \epsilon_t$, α should be zero. Effectively all CAPM tests have $H_0 : \alpha = 0$, $H_1 : \alpha > 0$, for all α of all portfolios jointly.

1.1 Gibbons, Ross, and Shanken

1. Compute the sharpe ratio $\hat{S}_q$ of the best ex-post portfolio q.
2. Compute the market sharpe ratio $\hat{S}_m$
3. Run test $T\frac{\hat{S}_q^2 - \hat{S}_m^2}{1+\hat{S}_m^2}$, where T is the number of time steps.

1.2 Fama-MacBeth Test

Tests if risk premia are what you hope them to be. Allows you to test **any factor model**.

1. Estimate $\hat{\beta}$ through $R_{pt} = k_t + \beta R_{mt} + \epsilon$.
2. Holding $\hat{\beta}$ fixed, estimate λ_t, $R_{pt} = \hat{\alpha}_t + \hat{\beta}\lambda_t + \epsilon$
3. $\hat{\lambda}$ is the mean of λ_t, same goes for $\hat{\alpha}$
4. Test that $\hat{\lambda} > 0$ and $\hat{\alpha} \neq 0$, using a simple t statistic
5. Econometricians like to add a "Shanken Correction" but this is a minor change

1.3 Issues in CAPM testing

1. **Roll Critique** It is not entirely clear what the *market portfolio* is. Any portfolio used for testing is just a proxy. So we might just be testing for a good proxy.

2. **Leverage constraints** lead to flattening SML,

3. **Short selling constraints** can generate downward sloping SML, further effects unclear.

4. **Dynamic risk**, β might not be constant over time

5. **Dynamic risk premium**, λ shown not be constant over time

1.4 Dynamic CAPM model

Extension to CAPM that allows for varying β and risk premia. Effectively just requires CAPM to be extended by one factor.

$$E[R_{pt}] = \gamma_0 + \bar{\beta}_p \gamma_1 + \beta_{prem,p} Var(\gamma_{1,t}) \tag{1}$$

Where γ_1 is a risk factor estimated from credit spreads. This model solves some of the ailments of CAPM, but α still exists.

1.5 Announcement Time Effect

1.5.1 CAPM and announcement days

The market proxy we observe is not the "true" market. Investors have private information about what the true factor is. Because all betas must sum to 1 (perfectly explain the market) an econometrician will not estimate the true beta. He will overestimate the beta of high "true beta" stocks and underestimate the beta of low "true beta" stocks. True beta is only revealed on specific days.

1.5.2 Announcement Risk

Firms that make scheduled (e.g. earnings) announcements earn excess returns. This is because the announcement contains some firm specific and some market news. Investors can not distinguish between the two and attribute too much of the market risk to the announcing firm. This effect is stronger for firms that announce earlier.

1.5.3 More fun puzzles

Option implied bound for equity risk premium. It can be shown that (theoretically) $E[R_t] - R_{f,t} \geq var^*[R_t]/R_{f,t}$. In practice, this is violated.

Dividend strips buy futures on 1 year forward dividends. These dividend strips have high alpha and low beta. Not entirely clear why (estimation error? Limited arbitrage?)

Human forecasters are not good, survey results show that expert human forecasts are too correlated with past returns and negatively correlated with future returns.

2 Multi Factor Models

Why have one factor if you can have many? **Economic approach** find persistent anomalies where securities with a certain trait consistently over or under perform. Hope there is a causal reason for that trait explaining returns. Factors should be uncorrelated, we are looking for the **optimal subset** of factors that helps us best explain returns for a broad set of securities. Factors could also be **misspricing**, suggesting that it is hard to arbitrage against them or **unmodeled risk**, existing factors that are missing in the model or **data mining** suggesting they only spuriously exist in sample.

2.1 Fama French 3-Factor

Constructs portfolios by sorting firms by size and book to market ratio. Arrives at 3 factors:

1. **Market**, CAPM portfolio return
2. **Size**, Average difference in returns on small-cap and large-cap firms
3. **Value**, Average difference in returns on high book to market and low-BM firms

Performs very well on Gibbons, Ross, and Shanken test. $\alpha \neq 0$ in small, low BM stocks. Does not explain momentum effect. Since FF3, many more models with many different factors.

3 Testing market efficiency

Market efficiency means that stock prices are unpredictable. There are three forms. We can test market efficiency over short and long horizons. Short-term return predictability is easy to detect if it is present, and hard to explain using a risk-based asset pricing model, because the risks would not manifest over such short periods. Long-term return predictability can have large effects on prices; harder to detect without a very long time series.

3.1 Weak form efficiency

Past returns predict future returns. This can be tested through an autocorrelation test. The **Ljung-Box** test for autocorrelation or Cochrane's variance ratio statistic can test autocorrelation. Result: Strong **cross correlation**, weak (negative) **autocorrelation** for individual stocks. Long run results much less clear. **Relative performance** r_t/σ_t also exhibits momentum.

3.2 Semi-Strong Form Efficiency (Stambaugh getting the stats right)

Large number of factors considered. Getting the statistics right is harder than you might think, **residuals are correlated, OLS yields biased estimates**.

$$r_t = \alpha + \beta x_{t-1} + \epsilon_t \tag{2}$$
$$x_t = \phi + \rho x_{t-1} + \mu_t \tag{3}$$
$$\epsilon_t = \gamma \mu_t + \nu_t \tag{4}$$

If we estimate β and ρ using OLS,

$$E[\hat{\beta} - \beta] = \gamma E[\hat{\rho} - \rho] \tag{5}$$

Stambaugh finds $E[\hat{\rho} - \rho] \approx -(1 + 3\rho)/T$, and uses this to de-bias estimate. He finds that this adjustment **clearly reduces the measured predictability, assuming no bubbles,** $\rho < 1$.

3.3 Goyal and Welch getting the stats right

G&W use a recursive regression in which they estimate factors from $t = 1, ..., \tau$ to predict returns in $\tau+1$. They benchmark their regression against a recursively computed mean return. They use this slightly adjusted measure to compare squared errors of mean and forecast $\Delta RMSE = \sqrt{SSE(M)/T} - \sqrt{SSE(R)/T}$. Results differ from classic R^2 result.

3.4 Significance of predictability

If an investor observes a forecast x_t with error ϵ, she can scale her investment and weight the security ω

$$\omega = \frac{\mu + x_t}{\gamma \sigma_\epsilon^2} \tag{6}$$

Since $R^2_{OOS} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\epsilon^2}$, the expected weighted return $E[\omega_t r_{t+1}]$ becomes:

$$E[\omega_t r_{t+1}] = \frac{1}{\gamma} \frac{S^2 + R^2_{OOS}}{1 - R^2_{OOS}} \tag{7}$$

Where S^2 is the security Sharpe ratio. The ratio of returns with or without prediction becomes $\approx \frac{R^2_{OOS}}{S^2}$ **if time horizons are short and R^2_{OOS} is relatively small**

4 The excess volatility puzzle

Stock prices are much more volatile than dividends. This was long a puzzle, but now it seems solved. We can derive a solution as follows:

1. Set up return formula $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$
2. Solve for P_t/D_t
3. Take logs
4. Take tailor approximation
5. Simplify so that $p_t - d_t \approx const + \sum_{j=0}^{\infty} \rho^j (\Delta d_{t+j+1} - r_{t+j+1}$ where Δd_{t+j+1} is the difference of log dividends between periods.
6. Now find that $var(p_t - d_t) = covar(p_t - d_t, \sum_{j=0}^{\infty} \rho^j \Delta d_{t+j+1}) - covar(p_t - d_t, \sum_{j=0}^{\infty} \rho^j r_{t+j+1})$.
7. Thus the price dividend ration **can only vary if it forecasts either dividend growth or returns**.

Almost all variation in P/D ratios is due to covaration with returns. Cashflows are not really predictable.

4.1 Decomposing returns

From the same equation we use to explain the volatility puzzle, we can derive an equation for unexpected returns.

$$r_{t+1} = E_t[r_{t+1}] \approx N_{CF,t+1} + N_{DR,t+1} \tag{8}$$

Where N stands for the difference in expectation between period t and $t+1$. We can estimate this with a VAR. Empirically **discount rate news and cashflow news are correlated**, leading to an under reaction of good cashflow news (as the demanded risk premium also rises). Cashflow news risk can be diversified, return risk less so. Using a CAPM on cashflows (not absolute returns) improves CAPM model a lot.

5 Zero Coupon Bonds

5.1 Terminology

ZCBs do not pay coupons (dividends) but only one fixed payment at maturity. The **yield to maturity** (YTMs) Υ_{nt} at time t of a bond with maturity n and price P_{nt} is defined as the bonds IRR.

$$P_{nt} = \frac{1}{(1 + \Upsilon_{nt})^n} \tag{9}$$

The plot of YTMs of bonds with different maturities is shown in a **yield curve**. THe **yield term spread** is $\Upsilon_{nt} - \Upsilon_{1t}$ where Υ_{1t} is the short rate (1 month). The **holding period return** is the return from holding a bond for one period and the ***excess* holding period return** is the holding period return less the short rate. All of the above are usually calculates in *logs*. The **forward rate** f_{tn} is the interest rate we can achieve by buying a ZCB with maturity $n+1$ and shorting an ZCB with maturity n.

5.2 Pure expectation hypothesis

Expected excess returns on bonds are zero. Long-term yields equal the average expected short-term yields over the corresponding period (aka a long term bond is the same as rolling over short term bonds).

5.3 Expectation hypothesis

Excess returns are constant over time. $E[r_{n,t+1} - y_{1,t}] = \mu(n)$ where μ is a risk premium and excess return depending only on the maturity. Unconditional tests confirm this hypothesis.

5.4 Conditional tests of expectation hypothesis

Campbell & Shiller: Under EH, yield depends on the yield term spread (the risk compensation for longer holding periods. This can be tested in a regression and turns out not to be true.

Alternative Specification: EH means that the yield of a long bond is the average of yields of short bonds. Todays bond yield must thus forecast future yields. This can be tested with a regression and turns out not to be true.

5.5 Term Structure Models: Vasicek

The price P_{1t} of a short bond is the expectation of the future discount factor M_{t+1}. The price of longer bonds are the autocovariance of the future stochastic discount factors.

$$P_{1t} = E_t[M_{t+1}] \tag{10}$$
$$P_{2t} = E_t[M_{t+1}M_{t+2}] \tag{11}$$

The model proposes a state variable z_t that describes the state of the world and has long run mean θ. z_t is seen to equal the short rate. The log SDF m_{t+1} is predicted from z_t with a shock:

$$-m_{t+1} = \frac{\lambda^2}{2} + z_t + \lambda\epsilon_{t+1} \tag{12}$$
$$z_{t+1} = (1-\varphi)\theta + \vartheta z_t + \sigma\epsilon_{t+1} \tag{13}$$

Where λ captures risk premia. A simple AR(1) model. θ, φ, σ can be estimated with an AR estimator. λ can be manually tuned to fit the data.

We also assume the log price of a ZCB is linear in z_t

$$p_{nt} = -A_n - B_n z_t \tag{14}$$

ZCBs with zero maturity have a price of 1 and a log price of 0. Thus $A_0 = B_0 = 0$.

5.5.1 Limitations of Vasicek

1. Insufficient curvature in mean spot rates
2. Allows negative interest
3. Constant volatility
4. Does not model spot rate (only yield curve resulting from a given rate)