Economics

MFE Exam Preparation Notes

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Macro Economics

MFE Midterm Notes

Macro 101

- Impulses hit the economy — E.g. technical breakthroughs
- They propagate through the economy — E.g. changing productivity
- And cause fluctuations in data — E.g. changing GDP
- We want to disentangle the impulses

Macro 102

Markets are connected and in a general, joint equilibrium

Changes in one market propagate to other markets

E.g. higher productivity -> Higher demand for labor -> Higher wages -> Higher consumption -> Higher prices for some goods

We need to disentangle:

- Trends
- Business Cycles
- Seasonality
- Random fluctuations
- Measurement error

How we define good models

- There are a number of facts that have been observed in the economy for a long time, we want our models to be able to express these observations:
  - Output per worker grows at a roughly constant rate
  - Capital per worker grows over time
  - Capital/output ratio is roughly constant
  - Rate of return to capital is constant
  - Shares of capital and labor in net income are nearly constant
  - Real wage grows over time
  - Ratios of consumption and investment to GDP are constant

Some further long term observations

- While not strictly binding, the model should also display these traits:
  - Non-durable consumption less volatile than output
  - Volatility of output and hours similar
  - Employment more volatile than average hours
  - Wages less volatile than productivity
  - Productivity slightly procyclical
  - Wages acyclical

Tool: Hodrick-Prescott Filter

Trend should be smooth but follow data closely

Squared Error Between Trend & Data
Deliver good predictions

Squared difference between trend deltas
Deliver stable trends

$$\min_{\{\tau_t\}_{t=1}^{T}} \left(\sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \right)$$

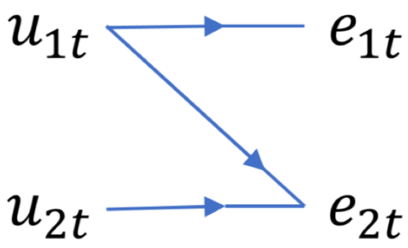
- Data consists of a trend and a cycle
- Estimate a trend which fits data closely but which also has stable deltas
- Parameter λ regulates tradeoff between smoothness and fit

Tool: Vector Autoregression (VAR)

- Regress variable X_t from previous data $X_{t-1}, X_{t-2}, \ldots, X_{t-n}$
- The length n of the lookback is often expressed with VAR(n)
- Utilizes standard OLS linear regression

The issue of ordering

- Errors in VAR should result from fundamental shocks
- But we can not clearly identify what influences what
- Causality can not be established
- By fixing some regression coefficients to zero, we can remove connections and regress only expected causal influences



VAR residuals and fundamental shocks

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} = \begin{pmatrix} \theta_1 & 0 \\ \theta_3 & \theta_4 \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \sim N\left[0; \begin{pmatrix} \sigma_1 & \sigma_{12} \\ \sigma_{12} & \sigma_2 \end{pmatrix}\right]$$

Unique identification $\sigma_1 = \theta_1^2$, $\sigma_{12} = \theta_1\theta_3$, $\sigma_2 = \theta_3^2 + \theta_4^2$

Sign Restrictions and Lambda Order

- Getting the order right is hard
- We can try out many orderings by multiplying the coefficient matrix with a rotation matrix
- We restrict the rotation parameter λ, to values that satisfy expectation of shock direction

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} = \begin{pmatrix} \theta_1 & 0 \\ \theta_3 & \theta_4 \end{pmatrix} \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

- Distribution of residuals

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim N\left[0; \begin{pmatrix} \theta_1^2 & \theta_1\theta_3 \\ \theta_1\theta_3 & \theta_3^2 + \theta_4^2 \end{pmatrix}\right]$$

for all $\lambda \in [-\pi, \pi]$

Simple General Equilibrium Model (SGE)

- Only households (no firms, production, etc.)
- Households choose between consumption and utility
- No carry forward utility (intertemporally separable utility)
- Households receive known endowment (aka income)
- Savings earn fixed interest rate
- Households maximize total utility subject to budget constraint and transversality conditions (next slides)

SGE Household Budget Constraint

Present value of savings = Present value of endowment less consumption + initial savings

Savings = Old savings with interest + endowment - consumption

$$a^i_{t+s+1} = R_{t+s} a^i_{t+s} + y^i_{t+s} - c^i_{t+s} \text{ for } \forall s \geq 0$$

SGE Transversality Condition

- Final savings must non-negative
- Present value of consumption must equal present value of endowment plus initial savings
- Rules out explosive borrowing in which all agents always borrow (Ponzi scheme)

$$\sum_{t=0}^{\infty} \frac{c_t^i}{\tilde{R}_t} = a_0^i + \sum_{t=0}^{\infty} \frac{y_t^i}{\tilde{R}_t}$$

$$\lim_{T\to\infty} \frac{a_T^i}{\tilde{R}_T} \geq 0$$

Euler Equation

- Beta = Discount rate
- R = Interest rate
- U = Utility
- Rewritten as
- $\beta R_{t+1} U_{c^i,t+1} = U_{c^i,t}$
- Marginal utility of consuming today is the same of marginal utility of consuming more (through interest) tomorrow after discounting by time value

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

SGE General Equilibrium

- If Log Utility, Euler eq. simplifies and we can substitute Utility with 1/consumption: $\sum_i c^i_{t+1} = \beta R_{t+1} \sum_i c^i_t$
- If market clearing & no aggregate savings: Aggregate consumption equals aggregate endowments
- Discounted market interest rate = average consumption tomorrow over average consumption today $\beta R_{t+1} = \frac{\sum_i y^i_{t+1}}{\sum_i y^i_t} = \frac{\bar{y}_{t+1}}{\bar{y}_t}$
- Because all agents are the same, these equations hold and the General Equilibrium Model Behaves like a representative agent model

Overlapping Generations & Rep. Agent Model

- Production economy with labor as only production driver
- Household level but no aggregate savings
- No capital accumulation for firms (100% depreciation)
- Log Utility
- Exogenous Labor Supply
- Transversality holds
- Overlapping Generations (OLG): Young agents work and safe, old agents live of their savings, savings chancel each other out.
- Representative Agent: One immortal agent maximizing utility

OLG Model

- Work when young, earn income, save and consume
- When old, consume savings
- Households maximize total discounted log utility
  - Euler equation describes optimal consumption
- Firms maximize production - wage costs – cost of capital

$$\max_{K_t, L_t} \left(K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t \right)$$

Alpha is use of capital in production

- Substituting consumption in Euler eq. the equilibrium is:

$$K_{t+1} = \frac{\beta}{1+\beta} (1-\alpha) K_t^\alpha$$

OLG with Tech Change

- Add productivity variable theta
- theta grows at constant rate g
- Multiply all labor inputs with theta
- New Equilibrium

$$K_{t+1} = \frac{\beta}{1+\beta}\left(1-\alpha\right) K_t^{\alpha}\theta_t^{1-\alpha}$$

- Normalized Equilibrium: Divide K by theta

Ramsey Representative Agent Model

Instead of old/young agent there is only one, immortal agent

Besides notation, nothing changes

Video: https://youtu.be/EHL1X0jV0dI

Endogenous Growth Models

Extended Accumulation Models (e.g. learning by doing)

Productivity is function of available capital

Innovation Models

Productivity is function of endogenous variables (e.g. R&D)

Real Business Cycle Model

Technology is uncertain, wages depend on tec, thus wages are uncertain

$$\max_{\{L_{t+s},C_{t+s},K_{t+s+1}\}} E_t \sum_{s=0}^{\infty} \beta^s \left(\log C_{t+s} - \chi L_{t+s}\right)$$

Chi L = Disutility of labor

- Ramsey w. endogenous labor supply
- Technology has trend component + random shocks
- Due to technology uncertainty, everything is in expectation
- Households derive utility from consumption and disutility from labor
- Euler eq. for consumption same as before
- Euler eq. for labor balances disutility from labor with wage gained from consumption
- By Transversality condition labor is constant

Simulating RBC

SET ALPHA, BETA & CHI AS FIXED TO OBTAIN REALISTIC NUMBERS

DRAW RANDOM TECHNOLOGY SHOCKS

COMPUTE TECH LEVEL, OUTPUT, ACCUMULATED CAPITAL ETC.

CAN ALSO DO IT THE OTHER WAY AROUND, ESTIMATE TECH SHOCKS FROM DATA

A Generalized RBC Model

01

Relax 100% depreciation rule

02

Labor supply no longer fixed

03

Labor supply increases today if tomorrows wages are expected to fall

- An upwards tech shock briefly increases labor supply through increased wages
- "Make hay while the sun shines"

04

Extra fancy: Infer alpha from data with Bayesian software

05

Problems: Hours worked not volatile enough, productivity too procyclical

Hansen Indivisible Labor

Instead of everyone adjusting hours, a share of the population works full time and other not at all

Households of working & non working members insure each other

Fixes volatility of hours worked but not procyclic hours worked

Labor Search

Economics invents applications and interviews

Chance of finding a job depends on labor market tightness

If unemployment is low, residual unemployed have hard time to find job because jobs are already taken

If unemployment is high, it is easier to find a job

Why hold money?

- Models discussed so far provide no incentive to have money
- Money (cash) is different from capital (investments in assets)
  - Money does not provide dividends
  - Money does not enter any production function
- Yet, people try to have some cash in the bank
- Fix 1: Add money to utility function
- Fix 2: Tie consumption to cash in previous period (cash in advance)
- Alternative: Remodel the whole economy: 3-equation model

Money in Utility (MIU)

- Add real money holdings to utility (money / price level), just like consumption
- Nominal: Price level affects capital and consumption prices as well

$$\max_{\{L_{t+s}, C_{t+s}, m_{t+s+1}, K_{t+s+1}\}} E_t \sum_{s=0}^{\infty} \beta^s \left(\log C_{t+s} + \log m_{t+s+1} - \chi L_{t+s} \right)$$

- Real: Price level replaced by discount rate which does not affect capital & consumption

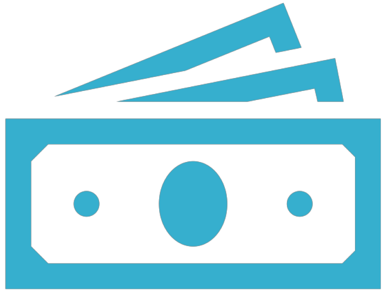
$$\max_{\{L_{t+s}, C_{t+s}, M_{t+s+1}, K_{t+s+1}\}} E_t \sum_{s=0}^{\infty} \beta^s \left(\log C_{t+s} + \log \frac{M_{t+s+1}}{P_{t+s}} - \chi L_{t+s} \right)$$

M = Money Balance
P = Price Level
C = Consumption
Chi = Disutility of Labor
L = Labor

- Captures various motivations to have money without modeling anything specific

MIU II

FOCs play out as usual, money does not affect equilibrium



By changing money supply we can modify all money related variables (e.g. price levels) but not real values of economy in equilibrium (labor, real wages, etc.)

Cash in Advance (CIA) Model

m = Real Money
C = consumption
pi = discount rate

$$m_{t+s} \geq C_{t+s}(1 + \pi_{t+s})$$

- Households have to have money for consumption
- Adds an extra household constraint
- Inflation effectively becomes tax on consumption
  - Pushes people to work less and consume untaxed leisure time
  - Pushes people to save more
- Changes to monetary policy (and thus to inflation) affects real economy variables
- Although influence of money is tiny (1% money supply increase = 0.04% output increase)

3-Equation Model

$$\frac{M_{t+1}}{P_t} = Y_t i_t^{-\eta}$$

Money supply over price level equals output times interest

- Instead of holding capital K, households buy government bonds
- Instead of allowing price level changes immediately, make prices "sticky"
- Money supply determined by demand for money

3 Equation Model: Notation Changes

$$\max_{\{C_t, N_t, B_t\}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

N = Labor
C = Consumption
σ, ϕ = Coefficients

s.t.

P = Price Level
B = Bond Holdings
Q = Bond Price
W = Wage
N = Labor
T = Transfers (endowments)

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

Output Gap

- Because prices are sticky, economy runs suboptimal
- Output gap: Output if firms can't change prices minus output if they could $\tilde{y}_t = y_t - y_t^n$
- Because there is no capital in the model, output (yt) equals consumption which equals solution of household problem

σ = Consumption Coefficient
i = Interest rate
E pi = expected future discount
rho = - log beta = inverse future discount

E y = Expected future output

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right)$$

Calvo Price Rigidity

- Probability theta that firm can't change prices
- Share of (1- theta) of firms change prices
- Firms that can change price to p*

$$p_t^* - \underbrace{p_{t-1}}_{\text{Old Price}} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \underbrace{\mu}_{\text{Desired Markup}} + \overbrace{mc_{t+k|t}}^{\text{Marginal Cost}} + (p_{t+k} - p_{t-1}) \} \quad \text{(G3.11)}$$

- Discount rate and prices follow

Inflation = price inelastiticty times price delta

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1})$$

$$\pi_t = \underbrace{\beta E_t \pi_{t+1}}_{\text{Discounted expected future inflation}} + \lambda \underbrace{(\mu + mc_t)}_{\text{Desired Markup + Marginal Cost}}$$

New Philips Curve

Current inflation is the discounted expected future inflation plus k times the output gap

$$\pi_t \quad = \quad \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

$$\kappa \quad \equiv \quad \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$$

lambda = langrarian see slide 31
alpha = use of capital
sigma = consumption coefficient

Taylor Rule

Interest — Inflation — Output Gap

$$i_t = 1 + 1.5\pi_t + 0.5\tilde{y}_t$$

- Constants inferred from data
- Works in “normal times”
- Since 2009: Not so much
- Banking our econ models on 30 years of “working data” when it does not work since 10 years: YOLO

3 Equation Model Extensions

- MONETARY POLICY, TECHNOLOGY, DEMAND & COST SHOCKS
- EXTEND TO OPEN ECONOMIES
- BASE TAYLOR RULE ON DOMESTIC INFLATION OR CPI
- INTRODUCE EXCHANGE RATES BETWEEN CURRENCIES

Friction Models

- Introduce bankers who intermediate between workers with savings and firms in which capital can be invested
- Banker promises households a fixed return R
- Banker receives a random return Rk
- Banker tries to maximize profit = Rk – R
- Bankers and workers perfectly insure each other (same household with shared consumption -> Communist utopia)
- Households invest with banker to maximize utility -> Plain old Euler eq.
- Frictionless equilibrium: R = Rk, basically the same as if banker was not there

Friction 1: Moral Hazard

- If Rk < R, the banker would make a loss
- Instead, the banker might choose to bankrupt and bail with a share of the managed money
- To prevent this, the banker needs to manage enough of his own money.
- Lost own money in case of bail needs to exceed loss from poor market result
- Add no default to equilibrium condition
- Banker promises lower returns R, fewer households invest, equilibrium is worse than without moral hazard
- The higher banker net worth, the more efficient the system

Friction 2: Moral Hazard & Effort

- Banker can exert effort to buy good securities
- The effort can not be monitored or controlled from the outside
- Effort is costly, reduced banker profits
- Households fully diversified across bankers by investing in them through diversified mutual funds
- Bankers face cash constraint, they can only pay as much as they have
- Cash constraint only binds if banker net worth is low
- In “normal times” (high net worth) banker is incentivized to exert high effort to take care of own high net worth (skin in the game)
- In “abnormal times” (low net worth) banker has “nothing to loose” and will thus not exert enough effort, leading to poor outcomes

Friction 3: Adverse Selection

- Households consist of bankers & workers
- Banker can invest in risky projects but need to borrow from mutual funds
- Banker pays a fixed borrowing rate
- Otherwise, banker can invest in mutual fund and earn fixed rate
- Risky project pays random amount theta with chance p
- Households and bankers know project outcome, mutual funds know distribution of outcomes
- Mutual funds make zero profit
- If bankers have low net worth and need to borrow much, not enough money is invested in risky projects: Socially not optimal

Friction 4: Asymmetric Information

- Bankers can fake bankruptcy (similar to moral hazard)
- Mutual funds need to spend on monitoring bankers
- If banker bankrupts, fund can recover a share of assets
- If that share goes down, returns on saving increase thus bankruptcy becomes more likely. Consumption & welfare decreases
- Higher probability of bankruptcy incentivizes banker to take more loans
- If banker has high net worth, this effect is reduced

Bank Runs

- Households invest in period 0, impatient withdraw at period 1, patient withdraw in period 2
- If only impatient withdraw in period 1, allocation is socially optimal
- Impatient withdrawing in period 1, patient in period 2 is a Nash equilibrium
- As soon as some patient withdraw in period 1, all patient are better off withdrawing in period 1
- Not enough funds to service all withdraws, some get nothing
- Everyone withdrawing in period 1 is a Nash equilibrium
- Because system has two Nash equilibria, it can “flip” from one to the other
- Fix: Tax withdraws in period 1. Tax depends on # consumers withdrawing to discourage patient to withdraw. Tax can insure deposits in period 2
- Kareken-Wallace: Deposit insurance introduces moral hazard and is no good fix

Game Theory

MFE Final Notes

Game Theory 101

- **Players** play **strategies** that lead to **payoffs**
- Players choose their strategies **simultaneously**
- A strategy is **strictly dominant** if it delivers a better outcome than all other strategies, **no matter what other players do**
- Strategy A is a **best response** to B if A delivers the strictly best payoff given that the other player plays B
- **A Nash equilibrium** is a set of best responses

Some Terminology

First best outcome: What could be achieved is all information was known

Second best outcome: Best outcome given that not all information is known

Iterated strict dominance

Pick two strategies

Check which one strictly dominates

Eliminate the dominated (worse) strategy

Keep going until only one strategy is left

ISD will never eliminate a Nash equilibrium **but** not all strategies found by ISD are Nash equilibria

Mixed strategies

- Sometimes it is best to act randomly
- A player must be **indifferent** to mix and that means the **other player must be indifferent**
- Don’t mix in a strategy that the other player can easily exploit. Don’t mix if you have a dominant strategy

Dynamic Games

Game in which players (turn wise) can choose to end the game with some outcome or continue to play and wait for the next outcome.

Solved using **backwards induction**, go from end of game backwards and see what best choice of each player is

Leads to **subgame perfect equilibrium (SPE)**, that describes what players do at each turn

SPE's are Nash equilibria

- Extends classical game theoretic games
- Players have **types** (that influences their payoff function) that only they know
- Other players have **priors** assigning probabilities to a player type
- Players can **signal** their type to influence others priors

Bayesian Games

Dynamic Games With Imperfect Information

- Extends Dynamic Games, but a **player does not know which action the other player might have taken**
- Players choose given their current **information set**
- Beliefs are updated according to **bayes rule**
- **Perfect Bayesian Equilibrium (PBE)** consisting of strategies and beliefs
- Every **PBE** is an **SPE** and thus **Nash equilibria**

Reputation

Player A can get a better outcome if player B thinks A is “crazy”

SPE might be suboptimal, so escaping SPE can lead to better outcome

A has to pass on a few “rational” decisions to build up a reputation

Eventually, A takes the rational choice and pops the bubble

If both players do this, a “rational bubble” can exist

Signaling

Players can **signal** their type by taking an action that mostly benefits the signaled type

A **pooled** BPE is when two types always signal the same type to be better off overall

Intuitive criterion the player benefitting more from signaling the other type will signal the other type. E.g. weak players will signal to be strong, not the other way around

Specian Signaling

- Signaling does not always lead to pooling
- If signal is too expensive or not worthwhile for one type it creates a separating equilibrium
- **Equilibrium dominance test:** Can a player improve its outcome by false signaling? If not, take signal at face value
- “Right type” has to overinvest to make signal believable

Repeated Games

- Play the same game over and over
- Leads to other SPEs as in the single game case
- Sequence of strategies represented by an **automaton**
- An automaton is SPE IFF no player ever has an incentive to deviate from it
- Repeated prisoners dilemma, **grim trigger** (cooperate until other defects, then always defect) is SPE
- **Noise** in observation greatly changes SPE
- Better to **forgive** because of possible false alarm
- **SPE is renegotiation proof** if no strategy pareto dominates

Collusion

For firms in a market, it is better to collude than to compete

But every firm has a short term incentive to break the cartel

Incentive is bigger with large future discount rate

Fast market growth and few firms make collusion easier

Other firms should punish diversion from cartel, punish, then forgive optimal

Noise problem demand observed by firm fluctuates naturally, firm not sure if others undercut

Punish then forgive optimal and observed strategy

Adverse selection

- Agents know their type
- Only “bad agents” opt into a deal
- Deal becomes unprofitable for good agents
- Spiral effect in which only bad agents remain and the market is underserved
- E.g. insurance is taken by the sick

Competitive Screening (in insurance)

- (Uninformed) firms offering deal make no profit
- **Pooling equilibrium** same deal accepted by low risk L and high risk H customers
- **Separating equilibrium** two deals that only get accepted by one group of customers
- The L indifference curve is steeper
- equilibrium, as indifference slopes of customer groups are different at common point and insurers are better off creating a new
- H types get full insurance, L types don't get full insurance, which is better for them and not envied by H types
- **Only L types will accept no full coverage**

Monopolistic Screening

- (Uninformed) firms offering deal are monopolies that maximize profits
- Monopoly offers contract (e.g. wage) that works out for the “bad” customer
- The “good” customers enjoy **information rent** by withholding information about their type
- Non-optimal outcome for everyone but worst customers

Cheap Talk

- Signaling involves taking a costly action
- Cheap talk -> No action taken
- Can still induce an action
- Can improve outcome

The Principal-Agent Problem

A principal hires and agent for a task. The agent exerts unobservable effort

If agent exerts effort, X% chance of positive outcome

First best benchmark if effort was observable, principal would pay wage that compensates for effort if the expected outcome from high effort is worth the cost

Second best (unobservable effort) solution pay conditionally on outcome. This leads to overall higher pay because of **agency cost**

The agent is induced to exert high effort by the incentive

The Holmström-Milgrom model

- Agent effort dominates outcome but there is **normally distributed noise**
- Agent has a CARA utility function wrt. its wage (Agent is risk averse)
- The optimal wage turns out to be linear wrt. the outcome
- Lots of extensions to account for e.g. multiple tasks
- Key result: The lower the noise, the more incentive based pay
- Competition pay only works if noise is correlated

Thanks for Reading

And Good Luck